STEADY TEMPERATURES DURING POROUS COOLING

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A mathematical model has been created in [1] for heat transfer processes with porous cooling, allowing for the temperature difference between the porous skeleton and the cooling agent. In [2] an investigation was made of this temperature difference as a function of the porosity of the wall, the internal heat transfer coefficient, and the Peclet number of the coolant. According to [2], the temperatures of the solid skeleton and of the coolant scarcely differ at any point of the body. A similar result was obtained in [3]. Thus, the model postulated in [4] is quite accurate. Some stationary problems have been examined in [5-10] on the basis of this model. In a recently published paper [11], problems were solved for three different bodies: an infinite plate, a thin-walled cylindrical tube, and a hollow sphere. The present paper examines the same problems, but for a thick-walled tube and sphere.

The coolant, coming from a reservoir at temperature t_0, passes into the body through $r = r_1$ ($r_1 = 0$ for the plate), and leaves at $r = R$ at a mass flow rate G_f, $\mathrm{kg/m^{2-\Gamma}\cdot hr}$, which is constant with time ($\Gamma = 0$, 1, and 2, respectively, for the plate, cylinder, and sphere). The amount of coolant passing per hour through the porous body has been referred for the cylinder to unit length of tubing, and for the plate—to unit surface area.

With the premises of reference [4] the temperature field is described by the equation

$$\frac{d^2t}{dr^2}+\frac{\Gamma}{r}\frac{dt}{dr}-\frac{q_f}{\lambda_{eq}}=0. \tag{1}$$

It is not difficult to show that the amount of heat absorbed by the coolant in unit time in unit volume of the body

$$q_f=-\frac{G_fc_f}{(2\pi\Gamma)}\frac{dt}{dr}, \tag{2}$$

where for $\Gamma = 0$ $(0) = 1$.

Following substitution of (2) into (1) in dimensionless form we obtain

$$\frac{d^2\Theta}{d\xi^2}+\left(\frac{\Gamma}{\xi}-\frac{g}{\xi^\Gamma}\right)\frac{d\Theta}{d\xi}=0. \tag{3}$$

The boundary condition on the internal surface when $\xi = \xi_1$ is

$$\frac{d\Theta(\xi_1)}{d\xi}=\frac{g}{\xi^\Gamma}\Theta(\xi_1). \tag{4}$$

We will consider that the heat supplied to the hot surface is expended not only in heating the body through conduction, but also in evaporation of the liquid. However, the heating is quite powerful, and therefore evaporation of the coolant proceeds only on the surface of the porous body.

For boundary conditions of the second kind, when $\xi = 1$, we obtain

$$\mathrm{Ki}=\frac{d\Theta(1)}{d\xi}+gK. \tag{5}$$

For boundary conditions of the third kind, when $\xi = 1$

$$\mathrm{Bi}\,|1-\Theta(1)|=\frac{d\Theta(1)}{g\xi}+gK. \tag{6}$$

The solutions of (3) are

$$\text{For } \Gamma=0 \quad \Theta=C_1\exp(g\xi)+C_2, \tag{7}$$

$$\text{For } \Gamma=1 \quad \Theta=C_1\xi^g+C_2, \tag{8}$$

$$\text{For } \Gamma=2 \quad \Theta=C_1\exp(-g/\xi)+C_2. \tag{9}$$

If the constants C_1 and C_2 are determined from (4) and (5), we obtain a solution of the problem for boundary conditions of the second kind

$$\Theta=\frac{\mathrm{Ki}-gK}{g}f_\Gamma(\xi), \tag{10}$$

where for the plate, tube, and hollow sphere $f_\Gamma(\xi)$

$$f_0(\xi)=\exp[-g(1-\xi)],\quad f_1(\xi)=\xi^g,$$

$$f_2(\xi)=\exp[g(1-1/\xi)]. \tag{11}$$

For the hot surface of the wall $\xi = 1$ and $f_\Gamma(1) = 1$.

If the constants C_1 and C_2 are determined from (4) and (6), we obtain a solution of the problem for boundary conditions of the third kind:

$$\Theta=\frac{1-g/\mathrm{Bi}\,K}{1+g/\mathrm{Bi}}f_\Gamma(\xi). \tag{12}$$

If the coolant is a gas, then $K = 0$. Then (10) for the plate gives the stationary part of the solution [12], while (12) for the plate goes over into the well-known expression [8]. From analysis of the solution of (10) and (12) the same conclusions follow which were drawn in [11].

NOTATION

r is the space coordinate; R is the characteristic dimension of the body; $\xi = r/R$; t, t_c and t_0 are the temperatures of porous body, heating medium, and coolant; $\Theta=(t-t_0)/(t_c-t_0)$; λ_{eq} is the equivalent thermal conductivity of porous body; c_f is the specific heat of the coolant; $g=G_fc_f\,R^{1-\Gamma}/(2\pi\Gamma)\,\lambda_{eq}$; α is the heat transfer coefficient; $\mathrm{Bi} = \alpha R/\lambda$; ρ is the heat of vaporization; $K=\rho/c_f\ (t_c-t_0)$; q_c is the given heat flux; $\mathrm{Ki} = q_cR/\lambda(t_c - t_0)$.

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